CBSE SAMPLE PAPER - 09

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\int_{-\pi}^{\pi} \sin^5 x dx = ?$ **[1]**

   a) $\frac{5\pi}{16}$ b) 2π

   c) 0 d) $\frac{3\pi}{4}$

2. Let P (α, β, γ) be any point on the plane x + 2y + 3z - 7 = 0. Then the least value of $\alpha^2 + \beta^2 + \gamma^2$ is equal to: **[1]**

   a) $\sqrt{\frac{7}{2}}$ b) 0

   c) 7 d) $\frac{7}{2}$

3. If the position vectors of P and Q are $\hat{i} + 3\hat{j} - 7\hat{k}$ and $5\hat{i} - 2\hat{j} + 4\hat{k}$ respectively , then the cosine of the angle between $\overrightarrow{PQ}$ and y-axis is **[1]**

   a) $\frac{4}{\sqrt{162}}$ b) $\frac{11}{\sqrt{162}}$

   c) $\frac{5}{\sqrt{162}}$ d) $-\frac{5}{\sqrt{162}}$

4. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is **[1]**

   a) $\frac{7}{32}$ b) $\frac{25}{192}$

   c) $\frac{25}{32}$ d) $\frac{1}{192}$

5. $\int \frac{x^2-1}{x^4+3x^2+1} dx$ is equal to **[1]**

   a) $\tan\left(x + \frac{1}{x}\right) + C$ b) $\tan^{-1}\left(x + \frac{1}{x}\right) + C$

   c) $\tan^{-1}\left(3x^2 + 2x\right) + C$ d) $\tan^{-1}\left(x^2 + 1\right) + C$

6. If A and B are such events that P(A) > 0 and P(B) $\neq$ 1, then P(A'/B') equals. **[1]**

a) $\frac{1-P(A\cup B)}{P(B')}$

b) $P(A')/P(B')$

c) $1-P(A/B)$

d) $1-P(A'/B)$

7. The area bounded by the parabola $y^2 = 4ax$, latus rectum and x-axis is **[1]**

a) 0

b) $\frac{2}{3}a^2$

c) $\frac{a^2}{3}$

d) $\frac{4}{3}a^2$

8. If the direction ratios of a line are proportional to 1, - 3, 2, then its direction cosines are **[1]**

a) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

b) $-\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}$

c) $\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

d) $-\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

9. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the expression $(\vec{a}-\vec{b})\cdot((\vec{b}-\vec{c})\times(\vec{c}-\vec{a}))$ equals. **[1]**

a) $[\vec{a}\vec{b}\vec{c}]$

b) $2[\vec{a}\vec{b}\vec{c}]$

c) None of these

d) $[\vec{a}\vec{b}\vec{c}]^2$

10. Find the equation of a curve passing through the point (0, 0) and whose differential equation is $y' = e^x \sin x$. **[1]**

a) $2y + 1 = e^x(\sin 2x - \cos x)$

b) $2y - 1 = (\sin x - \cos x)e^x$

c) $3y - 1 = e^x(\sin x - \cos 2x)$

d) $4y - 1 = e^x(\sin x - \cos 2x)$

11. If the area (in sq units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x, \lambda > 0$, is $\frac{1}{9}$, then λ is equal to **[1]**

a) 48

b) 24

c) $2\sqrt{6}$

d) $4\sqrt{3}$

12. $\int(\sin(\log x)+\cos(\log x))$ dx is equal to **[1]**

a) log (sin x - cos x) + c

b) x sin (log x) +C

c) sin (log x) – cos (log x) + C

d) x cos (log x) + C

13. The function $f(x) = x^2e^{-x}$ is Monotonic increasing when **[1]**

a) $x \in R - [0, 2]$

b) $0 < X < 2$

c) $2 < X < \infty$

d) $X < 0$

14. If A and B are symmetric matrices of the same order, then (AB′ –BA′) is a **[1]**

a) Skew symmetric matrix

b) Null matrix

c) Symmetric matrix

d) None of these

15. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if. **[1]**

a) $\lambda = 2$

b) $\lambda \neq -2$

c) None of these

d) $\lambda \neq 2$

16. For an invertible square matrix of order 3 with real entries $A^{-1} = A^2$, then det. A = **[1]**

a) $\frac{1}{3}$

b) 3

c) None of these

d) 1

17. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is **[1]**

a) None of these

b) $\frac{5\pi}{3}$

c) $\frac{\pi}{3}$

d) $\frac{2\pi}{3}$

18. y = 2 cos x + 3 sin x satisfies which of the following differential equations? **[1]**

i. $\frac{d^2y}{dx^2} + y = 0$

ii. $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} = 0$

Select the correct answer using the codes given below.

a) Only (ii)

b) Neither (i) nor (ii)

c) Only (i)

d) Both (i) and (ii)

19. **Assertion (A):** $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval (1, 2). **[1]**

Reason (R): $f'(x) < 0$ for $x \in (1, 2)$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If A = $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, then |A| = 0 **[1]**

Reason (R): $|\text{adj } A| = |A|^{n-1}$, where n is order of matrix.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$. **[2]**

22. If $\log\sqrt{\mathbf{x}^2 + \mathbf{y}^2} = \tan^{-1}\left(\frac{x}{y}\right)$, then show that $\frac{dy}{dx} = \frac{y-x}{y+x}$ **[2]**

23. A matrix A of order 3×3 is such that |A| = 4. Find the value of |2A|. **[2]**

OR

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find adj(AB)

24. Find the position vector (externally) of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2 : 1. **[2]**

25. A coin is tossed three times, determine P(E|F), **[2]**

where E: at least two heads, F: at most two heads.

Section C

26. Evaluate $\int_1^5 (|x-1| + |x-2| + |x-4|)dx$: **[3]**

27. Find the particular solution of the differential equation $x\frac{dy}{dx} - y + x\,cosec\left(\frac{y}{x}\right) = 0$. **[3]**

OR

Solve the initial value problem: $xy\frac{dy}{dx}$ = (x + 2) (y + 2), y(1) = -1

28. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that any two of them are non-collinear. If $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ **[3]**

and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$, then prove that $\vec{a}+2\vec{b}+6\vec{c}=\overrightarrow{0}$.

OR

Find the value of x such that the points A(3,2,1), B(4,x,5), C(4,2,-2) and D(6, 5,-1) are coplanar.

29. Evaluate $\int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{\frac{-x}{2}} dx$. **[3]**

OR

Evaluate the integral: $\int\limits_{0}^{\frac{\pi}{2}} \frac{\sin^2 x}{(1+\cos x)^2} dx$

30. If $\sqrt{y+x}+\sqrt{y-x}=c$, show that $\frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2}{x^2}-1}$ **[3]**

31. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line y = x in the first quadrant. **[3]**

Section D

32. Minimise Z = 400x + 200y ,subject to,$5x+2y \geqslant 30, 2x+y \geqslant 15, x \leqslant y, x \geqslant 0, y \geqslant 0$ **[5]**

33. Show that the function $f: R \rightarrow \{x \in R: -1 < x < 1\}$ defined by $f(x)=\frac{x}{1+|x|}$, x ∈ R is one-one and onto function. **[5]**

OR

Let A = R – {3} and B = R – {1}. Consider the function f: A → B defined by $f(x)=\left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

34. Find the length shortest distance between the lines: $\frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1}$ and $\frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}$ **[5]**

OR

A line with direction ratios (2, 2, 1) intersects the lines $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ and $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$ at the points P and Q respectively. Find the length and the equation of the intercept PQ.

35. Find the values of p and q so that $f(x)=\begin{cases} x^2+3x+p, if\, x \leqslant 1 \\ qx+2, if\, x>1 \end{cases}$ is differentiable at x = 1. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Mrs. Maya is the owner of a high-rise residential society having 50 apartments. When he set rent at ₹10000/month, all apartments are rented. If he increases rent by ₹250/ month, one fewer apartment is rented. The maintenance cost for each occupied unit is ₹500/month.

(i) If P is the rent price per apartment and N is the number of rented apartments, then find the profit.

(ii) If x represents the number of apartments which are not rented, then express profit as a function of x.

(iii) Find the number of apartments which are not rented so that profit is maximum.

OR

Verify that profit is maximum at critical value of x by second derivative test.

37. **Read the text carefully and answer the questions:** **[4]**

In a city, there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys, the number of units of types I, II, and III respectively are 80, 70, and 65 in factory A and 85, 65, and 72 are in factory B. For girls the number of units of types I, II, and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

(i) Represent the number of units of each type produced by factory A for both boys and girls and number of units of each type produced by factory B for both boys and girls in matrix form.

(ii) Find the total production of sports clothes of each type for boys.

(iii) Find the total production of sports clothes of each type for girls.

OR

Let R be a 3×2 matrix that represent the total production of sports clothes of each type for boys and girls, then find the transpose of R.

38. **Read the text carefully and answer the questions:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(i) Find the probability that both of them are selected.

(ii) The probability that none of them is selected.

Solution

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Section A

1. **(c)** 0

   Explanation: If f is an odd function,
   $\int_{-a}^{a} f(x)dx = 0$
   as, $\int_0^a f(x)dx = -\int_{-a}^{0} f(x)dx$
   f(x) = $\sin^5$ x
   f(-x) = $\sin^5$ (-x)
   Therefore, f(x) is odd number
   $\int_{-\pi}^{\pi} \sin^5 x dx = 0$

2. **(d)** $\frac{7}{2}$

   Explanation: Given that P(α, β, γ) lies on the plane x + 2y + 3z - 7 = 0
   $\Rightarrow \alpha + 2\beta + 3\gamma - 7 = 0$
   $\Rightarrow \alpha + 2\beta + 3\gamma = 7$ ... (i)
   $|ax + by + cz| \leq \sqrt{a^2 + b^2 + c^2}\sqrt{x^2 + y^2 + z^2}$
   $\Rightarrow |\alpha + 2\beta + 3\gamma| \leq \sqrt{1^2 + 2^2 + 3^2}\sqrt{\alpha^2 + \beta^2 + \gamma^2}$
   $\Rightarrow 7 \leq \sqrt{14}\sqrt{\alpha^2 + \beta^2 + \gamma^2}$
   $\Rightarrow \sqrt{\alpha^2 + \beta^2 + \gamma^2} \geq \frac{7}{\sqrt{14}}$
   $\Rightarrow \alpha^2 + \beta^2 + \gamma^2 \geq \frac{49}{14} \Rightarrow \alpha^2 + \beta^2 + \gamma^2 \geq \frac{7}{2}$
   $\Rightarrow$ Least value of $\alpha^2 + \beta^2 + \gamma^2 = \frac{7}{2}$

3. **(d)** $-\frac{5}{\sqrt{162}}$

   Explanation: Given position vectors $\overrightarrow{OP} = \hat{i} + 3\hat{j} - 7\hat{k}$ and $\overrightarrow{OQ} = 5\hat{i} - 2\hat{j} + 4\hat{k}$
   $\Longrightarrow$ drs of $\overrightarrow{PQ}$ =$\overrightarrow{OQ} - \overrightarrow{OP} = 4\hat{i} - 5\hat{j} + 11\hat{k}$ and drs along with y-axis are (0,1,0) 0r $\hat{j}$
   direction cosines between $\overrightarrow{PQ}$ and y-axis is $\frac{(4\hat{i} - 5\hat{j} + 11\hat{k}).\hat{j}}{\sqrt{16+25+121}} = \frac{-5}{\sqrt{162}}$

4. **(c)** $\frac{25}{32}$

   Explanation: Given that probability of hitting a target independently by four persons are respectively
   $P_1 = \frac{1}{2}, P_2 = \frac{1}{3}, P_3 = \frac{1}{4} and P_4 = \frac{1}{8}$
   Then, the probability of not hitting the target is
   $= \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)$
   [$\because$ events are independent]
   $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = \frac{7}{32}$
   Therefore, the required probability of hitting the target = 1 - (Probability of not hitting the target)
   $= 1 - \frac{7}{32} = \frac{25}{32}$

5. **(b)** $\tan^{-1}\left(x + \frac{1}{x}\right) + C$

   Explanation: Divide num. and deno. by x^2
   Substitute $x + \frac{1}{x} = t$ then $(1 - \frac{1}{x^2})dx = dt$
   $\Rightarrow \int \frac{dt}{t^2+1}$
   $\Rightarrow \tan^{-1}(x + \frac{1}{x}) + C$

6. **(a)** $\frac{1 - P(A \cup B)}{P(B')}$

   Explanation: $\because P(A) > 0$ and $P(B) \neq 1$
   $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{P(B')}$

7. **(d)** $\frac{4}{3}a^2$

Explanation: X - cordinate of latus rectum is a

$\Rightarrow \int_0^a ydx = \int_0^a 2\sqrt{a}\sqrt{x}dx$

$\Rightarrow \int_0^a ydx = 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]$

$\Rightarrow \int_0^a ydx = \frac{4a^2}{3}$

8. **(c)** $\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

Explanation: $\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

The direction ratios of the line are proportional to 1, -3, 2

$\therefore$ The direction cosines of the line are

$\frac{1}{\sqrt{1^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{1^2+(-3)^2+2^2}}, \frac{2}{\sqrt{1^2+(-3)^2+2^2}}$

$= \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

9. **(c)** None of these

Explanation: Given $\vec{a}, \vec{b}\ and\ \vec{c}$ are any three vectors

now, $(\vec{a}-\vec{b})\cdot((\vec{b}-\vec{c})\times(\vec{c}-\vec{a})]$

$=(\vec{a}-\vec{b})\cdot(\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{c}\times\vec{c}+\vec{c}\times\vec{a})$

$=(\vec{a}-\vec{b})\cdot(\vec{b}\times\vec{c}-\vec{b}\times\vec{a}+\vec{c}\times\vec{a})$ (where $\vec{c}\times\vec{c}=\vec{0}$)

$=\vec{a}\cdot(\vec{b}\times\vec{c})-\vec{a}\cdot(\vec{b}\times\vec{a})+\vec{a}\cdot(\vec{c}\times\vec{a})-\vec{b}\cdot(\vec{b}\times\vec{c})+\vec{b}\cdot(\vec{b}\times\vec{a})-\vec{b}.(\vec{c}\times\vec{a})$

$=[\vec{a}\vec{b}\vec{c}]-0+0-0+0-[\vec{b}\vec{c}\vec{a}]$

$=[\vec{a}\vec{b}\vec{c}]-[\vec{a}\vec{b}\vec{c}]$

10. **(b)** $2y-1=(\sin x-\cos x)e^x$

Explanation: $\frac{dy}{dx}=e^x\sin x$

$\int dy=\int e^x\sin x dx$

$y=\frac{1}{2}(\sin x-\cos x)e^x+C$

When x = y = 0 , we get

$0=\frac{1}{2}(\sin 0-\cos 0)e^0+C$

$C=\frac{1}{2}$

Hence, $y=\frac{1}{2}(\sin x-\cos x)e^x+\frac{1}{2}$

$2y-1=(\sin x-\cos x)e^x$

11. **(b)** 24

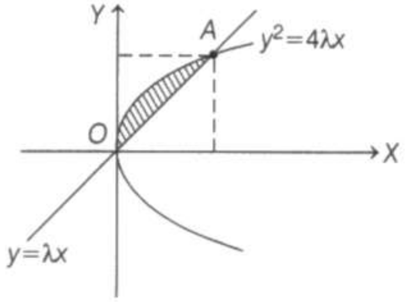
Explanation: Given, equation of curves are

$y^2= 4\lambda x$ ... (i)

and $y = \lambda x$ ...(ii)

$\lambda > 0$

Area bounded by above two curve is, as per figure



the intersection point A we will get on the solving Eqs. (i) and (ii), we get

$\lambda^2 y^2= 4\lambda x$

$\Rightarrow \quad x=\frac{4}{\lambda}$, so $y=4$

So, 4 $\left(\frac{4}{\lambda},4\right)$

Now, required area is = $=\int_0^{4/\lambda}(2\sqrt{\lambda x}-\lambda x)dx$

$= 2\sqrt{\lambda}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^{4/\lambda} - \lambda\left[\frac{x^2}{2}\right]_0^{4\lambda}$

$= \frac{4}{3}\sqrt{\lambda}\frac{4\sqrt{4}}{\lambda\sqrt{\lambda}} - \frac{\lambda}{2}\left(\frac{4}{\lambda}\right)^2$

$= \frac{32}{3\lambda} - \frac{8}{\lambda} = \frac{32-24}{3\lambda} = \frac{8}{3\lambda}$

It is given that area = $\frac{1}{9}$

$\Rightarrow \quad \frac{8}{3\lambda} = \frac{1}{9}$

$\Rightarrow \quad \lambda = 24$

12. **(b)** x sin (log x) +C

Explanation: $\int(sin(log\ x) + cos(log\ x))dx$

(Use By Part, Take 1 as II function)

$= \int \sin(log\ x).1dx + \int \cos(\log\ x)dx$

$= (\sin(\log x)).x - \int \cos(\log x)\frac{1}{x}.xdx + \int \cos(log\ x)dx.$

$= x\sin(\log x) + C$

13. **(b)** 0 < X < 2

Explanation: 0 < X < 2

$f(x) = x^2e^{-x}$

$f'(x) = 2xe^{-x} - x^2e^{-x}$

$= e^{-x}(2 - x)$

For f(x) to be monotonic increasing, we must have

$f'(x) > 0$

$\Rightarrow e^{-x}x(2 - x) > 0\ [\because e^{-x} > 0]$

$\Rightarrow x(2 - x) > 0$

$\Rightarrow x(x - 2) < 0$

$\Rightarrow 0 < x < 2$

14. **(a)** Skew symmetric matrix

Explanation: Given:(AB′ –BA′) = (AB′)′ – (BA′)′

= (AB)' - (BA)' ...(∵ A' = A and B' = B)

= B'A' - A'B'

= BA' - AB'

= - (AB' - BA')

Therefore, we obtain(AB′ –BA′) is Skew-Symmetric.

15. **(c)** None of these

Explanation: We have,

$$A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$$

A^{-1} exists if $|A| \neq 0$

Now $|A| = 2(6 - 5) - \lambda(-5) - 3(-2) = 8 + 5\lambda \neq 0$

$\Rightarrow 5\lambda \neq -8$

$\Rightarrow \lambda \neq \frac{-8}{5}$

So, A^{-1} exists if and only if $\neq \frac{-8}{5}$

16. **(d)** 1

Explanation: $A^2 = I \Rightarrow A^2A^{-1} = IA^{-1} \Rightarrow A = A^{-1}$ and it is possible only if A is an identity matrix and determinant of the identity matrix is equal to 1

17. **(c)** $\frac{\pi}{3}$

Explanation: Let $x = \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$\Rightarrow \sin x = \sin\left(\frac{2\pi}{3}\right)$

Here the range of principle value of sine is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow$ x = $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence, for all values of x in range $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$, the value of

$\sin^{-1}\left(\sin(\frac{2\pi}{3})\right)$ is

$\Rightarrow$ sin x = sin $\left(\pi - \frac{\pi}{3}\right)$ ($\because$ sin $(\frac{2\pi}{3})$ = sin $\left(\pi - \frac{\pi}{3}\right)$)

$\Rightarrow$ sin x = sin $\left(\frac{\pi}{3}\right)$ ($\because \sin(\pi - \theta) = \sin\theta$ as here θ lies in II quadrant and sine is positive)

$\Rightarrow x = \frac{\pi}{3}$

18. **(c)** Only (i)

Explanation: Given differential equation is

y = 2 cos x + 3 sin x ...(i)

Now, $\frac{dy}{dx}$ = = -2sin x + 3cos x

= -(2 cos x + 3 sin x) = -y [from Eq. (i)]

$\therefore \frac{d^2y}{dx^2}$ + y = 0

So, only Statement (i) is correct.

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: We have, f(x) = $2x^3 - 9x^2 + 12x - 3$

$\Rightarrow$ f'(x) = $6x^2 - 18x + 12$

For increasing function, f'(x) ≥ 0

$\therefore 6(x^2 - 3x + 2) \geq 0$

$\Rightarrow 6(x - 2)(x - 1) \geq 0$

$\Rightarrow x \leq 1$ and $x \geq 2$

$\therefore$ f(x) is increasing outside the interval (1, 2), therefore it is a true statement.

Reason: Now, f'(x) < 0

$\Rightarrow 6(x - 2)(x - 1) < 0$

$\Rightarrow 1 < x < 2$

$\therefore$ Assertion and Reason are both true but Reason is not the correct explanation of Assertion.

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: The given matrix is $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

Then, |A| = $\begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

By expanding along R_1 (first row), we get

$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

= 1(- 9 + 12) - 1 (-18 + 15) - 2 (8 - 5) = 1 (3) - 1(-3) - 2(3) = 3 + 3 - 6 = 0, which is a true statement.

Reason: |adj (A)| = $|A|^{n-1}$ is a true statement.

Hence, both assertion and reason are true but reason is not a correct explanation of assertion.

Section B

21. We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1)$.

$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right) \begin{bmatrix} \because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0, \pi) \\ and \tan^{-1}(-x) = -\tan^{-1}x \end{bmatrix}$

$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi + 4\pi - 3\pi}{12}$

$= \frac{-5\pi + 4\pi}{12} = \frac{-\pi}{12}$

22. $\log \sqrt{x^2+y^2} = \tan^{-1}\left(\frac{x}{y}\right)$

$\Rightarrow \frac{1}{2}\log\left(x^2+y^2\right) = \tan^{-1}\left(\frac{X}{y}\right)$

$\log\left(x^2+y^2\right) = 2\tan^{-1}\left(\frac{x}{y}\right)$

On differentiating both sides w.r.t. to x, we get,

$\frac{1}{x^2+y^2}\left[2x+2y\frac{dy}{dx}\right] = \frac{2}{1+\left(\frac{1}{y}\right)^2}\left[\frac{y-x\frac{dy}{dx}}{y^2}\right]$

$2\left[\frac{1}{x^2+y^2}\left[x+y\frac{dy}{dx}\right] = 2\left[\frac{y^2}{x^2+y^2}\left[\frac{y-x\frac{dy}{dx}}{y^2}\right]\right]\right.$

$\left[\frac{1}{x^2+y^2}\left[x+y\frac{dy}{dx}\right] = \frac{1}{x^2+y^2}\left[y-x\frac{dy}{dx}\right]\right.$

$\frac{dy}{dx}(y+x) = y-x$

$\frac{dy}{dx} = \frac{y-x}{y+x}$

Hence proved

23. We are given that,

Order of matrix A = 3

|A| = 4

We need to find the value of |2A|.

By the property of determinant of a matrix,

$|KA| = K^n|A|$

Where the order of the matrix A is n.

Similarly,

$|2A| = 2^3|A|$ ...[∵ Order of matrix A = 3]

⇒ |2A| = 8|A|

Substituting the value of |A| in the above equation,

⇒ |2A| = 8 × 4

⇒ |2A| = 32

Thus, the value of |2A| is 32.

OR

Here, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

adj(AB) = adjB.adjA

$\operatorname{adj} A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\operatorname{adj} B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\operatorname{adj}(AB) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Hence, $adj(AB) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

24. Given that $\vec{P} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} - \hat{\mathrm{k}}$ and $\vec{\mathrm{Q}} = -\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}$

The $\vec{\mathrm{R}}$ does not lie on the segment PQ (external division). If $\mathrm{m} : n$ is the ratio in which $\vec{\mathrm{R}}$ divides PQ, then

$\vec{\mathrm{R}} = \frac{m\vec{\mathrm{Q}} - n\vec{\mathrm{P}}}{m-n}$

Given m : n = 2 : 1, m = 2 and n = 1

$\Rightarrow \vec{\mathrm{R}} = \frac{2(-\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}) - 1(\hat{\mathrm{i}}+2\hat{\mathrm{j}}-\hat{\mathrm{k}})}{2-1} = \frac{-3\hat{\imath}+0\hat{\jmath}+3\hat{k}}{1} = -3\hat{\imath} + 3\hat{k}$

25. The sample space of the given experiment will be:

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Here, E: at least two heads

And F: at most two heads

⇒ E = {HHH, HHT, HTH, THH}

and F = {HHT, HTH, THH, HTT, THT, TTH, TTT}

⇒ E ∩ F = {HHT, HTH, THH}

So, P(E) = $\frac{4}{8}=\frac{1}{2}$, P(F) = $\frac{7}{8}$, P(E $\cap$ F) = $\frac{3}{8}$

By the definition of conditional probability $P(E|F)=\frac{P(E\cap F)}{P(F)}$

$\Rightarrow P(E|F)=\frac{\frac{3}{8}}{\frac{7}{8}}=\frac{3}{7}$.

Which is the required solution.

Section C

26. $\int_1^5|x-11+1x-21+1x-4|dx$

= $\int_1^2(5-x)dx+\int_2^4(x+1)dx+\int_4^5(3x-7)dx$

= $\left[5x-\frac{x^2}{2}\right]_1^2+\left[\frac{x^2}{2}+x\right]_2^4+\left[\frac{3x^2}{2}-7x\right]_4^5$

= $\frac{7}{2}$ + 8 + $\frac{13}{2}$ = 18

27. Given differential equation is

$x\frac{dy}{dx}-y+xcosec\ \left(\frac{y}{x}\right)=0$

On dividing both sides by x , we get

$\Rightarrow\quad \frac{dy}{dx}-\frac{y}{x}+cosec\ \left(\frac{y}{x}\right)=0$

$\Rightarrow\quad \frac{dy}{dx}=\frac{y}{x}-cosec\ \left(\frac{y}{x}\right)$

which is a homogeneous differential equation as $\frac{dy}{dx}=F\left(\frac{y}{x}\right)$.

put , $y=vx$

$\Rightarrow\quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx}=\frac{vx}{x}-cosec\ \left(\frac{vx}{x}\right)$

$\Rightarrow\quad v+x\frac{dv}{dx}=v-cosec\ v$

$\Rightarrow\quad x\frac{dv}{dx}=-cosec\ v\Rightarrow\frac{dv}{cosec\ v}=\frac{-dx}{x}$

On integrating both sides, we get

$\int\frac{dv}{cosec\ v}=\int-\frac{dx}{x}$

$\Rightarrow\int\sin vdv=\int-\frac{dx}{x}$

$\Rightarrow\quad -\cos v=-\log|x|+C$

$\Rightarrow\quad -\cos\frac{y}{x}=-\log|x|+c\quad\left[\text{put } v=\frac{y}{x}\right]$

$\Rightarrow\quad \cos\frac{y}{x}=(\log|x|-C)$ ....(i)

Also, given that x = 1 and y = 0.

On putting above values in Eq. (i), we get

$\Rightarrow\quad \cos 0=\log|1|-C$

$\Rightarrow\quad 1=0-C\Rightarrow C=-1$

$\therefore\quad \cos\frac{y}{x}=\log|x|+1$ [from Eq. (i)]

This is required solution of given differential equation.

OR

Given that,

$xy\frac{dy}{dx}$ = (x + 2) (y + 2), y(1) = -1

$\Rightarrow\frac{y}{y+2}dy=\frac{x+2}{x}dx$

$\Rightarrow\frac{y+2-2}{y+2}dy=\frac{x+2}{x}dx$

$\Rightarrow\left(1-\frac{2}{y+2}\right)dy=\left(1+\frac{2}{x}\right)dx$

Integrating both sides, we get

$\int\left(1-\frac{2}{y+2}\right)dy=\int\left(1+\frac{2}{x}\right)dx$

$\Rightarrow$ y - 2 log |y + 2| = x + 2 log |x| + C ...(i)

Given that y(1) = -1, i,e, at x = 1, y = -1

Substituting the values of x and y in (i), we get

-1 - 2 log |1| = 1 + 2 log |1| + C

$\Rightarrow$ - 1 = 1 + C

$\Rightarrow$ C = -2

Substituting the value of C in (i), we get

y - 2 log |y + 2| = x + 2 log |x| - 2

Hence, y - 2 log |y + 2| = x + 2 log |x| - 2 is the required solution.

28. It is given that

$\vec{a}+2\vec{b}$ is collinear with $\vec{c}$

$\Rightarrow \quad \vec{a}+2\vec{b}=\lambda\vec{c}$ for some scalar λ .... (i)

Also given,

$\vec{b}+3\vec{c}$ is collinear with $\vec{a}$

$\Rightarrow \quad \vec{b}+3\vec{c}=\mu\vec{a}$ for some scalar μ .... (ii)

From (i), we get

$\vec{a}=\lambda\vec{c}-2\vec{b}$

Substituting this value of $\vec{a}$ in (ii), we get

$\vec{b}+3\vec{c}=\mu(\lambda\vec{c}-2\vec{b})$

$\Rightarrow \quad (1+2\mu)\vec{b}+(3-\mu\lambda)\vec{c}=\overrightarrow{0}$

$\Rightarrow \quad 1+2\mu=0$ and $3-\mu\lambda=0$ [$\because \vec{b}$ and $\vec{c}$ are non-collinear vectors]

$\Rightarrow \quad \mu=-\frac{1}{2}$ and $\lambda=-6$

Substituting the values of λ and μ in (i) and (ii), respectively, in both cases we get $\vec{a}+2\vec{b}+6\vec{c}=\overrightarrow{0}$.

OR

According to the question ,

A= (3,2,1),

B= (4,x,5),

C =(4,2,-2) and

D = (6,5,-1).

Now ,

$\overrightarrow{AB}=\hat{i}+(x-2)\hat{j}+4\hat{k}$

$\overrightarrow{AC}=\hat{i}-3\hat{k}$ and

$\overrightarrow{AD}=3\hat{i}+3\hat{j}-2\hat{k}$

Since, $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar

$\therefore [\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]=0$

$$\Rightarrow \begin{vmatrix} 1 & (x-2) & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix}=0$$

$\Rightarrow \quad 1(0+9)-(x-2)(-2+9)+4(3-0)=0$

⇒ 9- (x - 2)(7) + 12 = 0

⇒ 9 - 7x + 14 + 12 = 0

⇒ 35 - 7x = 0

$\Rightarrow x=\frac{35}{7}=5$

29. Given, $I=\int \frac{\sqrt{1-\sin x}}{1+\cos x}\cdot e^{\frac{-x}{2}}\,dx$

Let $\frac{-x}{2}=t \Rightarrow dx=-2dt$

$I=\int \frac{\sqrt{1-\sin(-2t)}}{1+\cos(-2t)}e^{t}(-2dt)$ [$\because x=-2t$]

$=-2\int e^{t}\frac{\sqrt{1+\sin 2t}}{1+\cos 2t}dt$[$\because \sin(-\theta)=-\sin\theta$ and $\cos(-\theta)=\cos\theta$]

$=-2\int e^{t}\frac{\sqrt{cos^2x+sin^2x+2sinxcosx}}{1+\cos 2t}dt$[$\because cos^2x+sin^2x=1, sin2x=2sinxcosx$]

$=-2\int e^{t}\left(\frac{\sqrt{(\cos t+\sin t)^2}}{2\cos^2 t}\right)dt$[$\because (a+b)^2=a^2+b^2+2ab$]

$=-2\int e^{t}\left(\frac{\cos t+\sin t}{2\cos^2 t}\right)dt$

$=-\int e^{t}(\sec t+\tan t\sec t)dt$ [$\because \frac{1}{cosx}=secx, \frac{sinx}{cosx}=tanx$]

we know that , $\int e^{t}[f(t)+f'(t)]\,dt=e^{t}f(t)+C$

Now, consider $f(t)=sec\,t$

then $f'(t) = sec\, t\, tan\, t$

$\therefore I = e^t \sec t + C$

$= -e^{-x/2} \sec \frac{x}{2} + C \left[\because t = \frac{-x}{2} \text{ and } \sec(-\theta) = \sec\theta\right]$

$I = -e^{-x/2} \sec \frac{x}{2} + C$

OR

Let the given integral be,

$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{(1+\cos x)^2} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{\left(2\cos^2\frac{x}{2}\right)^2} dx \left[\because \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} \ \ and \ 1+\cos x = 2\cos^2\frac{x}{2}\right]$

$= \int_0^{\frac{\pi}{2}} \frac{\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{\cos^4\frac{x}{2}} dx$

$= \int_0^{\frac{\pi}{2}} \tan^2\frac{x}{2} dx$

Add and subtract 1 to the integrand, we get

$\int_0^{\frac{\pi}{2}} \left(1+\tan^2\frac{x}{2}\right) - 1 dx$

$= \int_0^{\frac{\pi}{2}} \left(\sec^2\frac{x}{2} - 1\right) dx \ \left[\because \int \sec^2\frac{x}{2} dx = 2\tan\frac{x}{2}\right]$

$= \left[2\tan\frac{x}{2} - x\right]_0^{\frac{x}{2}} = \left(2\tan\frac{\pi}{4} - \frac{\pi}{2} - 0 + 0\right)$

$= 2 - \frac{\pi}{2}$

$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{(1+\cos x)^2} dx = 2 - \frac{\pi}{2}$

30. Here, $\sqrt{y+x} + \sqrt{y-x} = c$

Differentiating with respect to x,

$\Rightarrow \frac{d}{dx}(\sqrt{y+x}) + \frac{d}{dx}\sqrt{y-x} = \frac{d}{dx}(c)$

$\Rightarrow \frac{1}{2\sqrt{y+x}} \frac{d}{dx}(y+x) + \frac{1}{2\sqrt{y-x}} \frac{d}{dx}(y-x) = 0$

$\Rightarrow \frac{1}{2\sqrt{y+x}}\left(\frac{dy}{dx}+1\right) + \frac{1}{2\sqrt{y-x}}\left(\frac{dy}{dx}-1\right) = 0$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{2\sqrt{y+x}}\right) + \frac{dy}{dx}\left(\frac{1}{2\sqrt{y-x}}\right) = \frac{1}{2\sqrt{y-x}} - \frac{1}{2\sqrt{y+x}}$

$\Rightarrow \frac{dy}{dx} \times \frac{1}{2}\left[\frac{1}{\sqrt{y+x}} + \frac{1}{\sqrt{y-x}}\right] = \frac{1}{2}\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$

$\Rightarrow \frac{dy}{dx}\left[\frac{\sqrt{y-x}+\sqrt{y+x}}{\sqrt{y+x}\sqrt{y-x}}\right] = \left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y+x}+\sqrt{y-x}} \times \frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y+x}-\sqrt{y-x}}$ [rationalising the denominator]

$\Rightarrow \frac{dy}{dx} = \frac{(y+x)+(y-x)-2\sqrt{y+x}\sqrt{y-x}}{y+x-y+x}$

$\Rightarrow \frac{dy}{dx} = \frac{2y-2\sqrt{y^2-x^2}}{2x}$

$\Rightarrow \frac{dy}{dx} = \frac{2y}{2x} - \frac{2\sqrt{y^2-x^2}}{2x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2-x^2}{x^2}}$

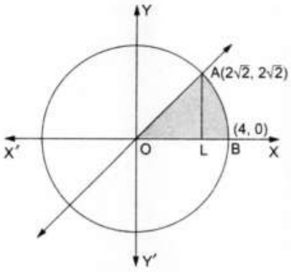
$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$

LHS=RHS

Hence Proved.

31. The given circle is $x^2 + y^2 = 16$ ...(i)

The given line is y = x ...(ii)



Putting y = x from (ii) into (i), we get

$2x^2 = 16 \Leftrightarrow x^2 = 8 \Leftrightarrow x = 2\sqrt{2}$ [$\because$ x is +ve in the first quad.}

Thus, the point of intersection of (i) and (ii) in the first quadrant is $A(2\sqrt{2}, 2\sqrt{2})$

Draw AL perpendicular on the x-axis

Therefore required area of region = (area of region OLA) + area of region(LBAL).

$= \int_0^{2\sqrt{2}} x dx + \int_{2\sqrt{2}}^{4} \sqrt{16-x^2} dx$

$= \left[\frac{x^2}{2}\right]_0^{2\sqrt{2}} + \left[\frac{x\sqrt{16-x^2}}{2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_{2\sqrt{2}}^{4}$

$= \frac{1}{2}[(2\sqrt{2})^2 - 0] + \left[\left(0 + 8\sin^{-1} 1\right) - \left(4 + 8\sin^{-1}\frac{1}{\sqrt{2}}\right]\right.$

$= \left[4 + \left(8 \times \frac{\pi}{2}\right) - 4 - \left(8 \times \frac{\pi}{4}\right)\right]$ = (2π) sq units.

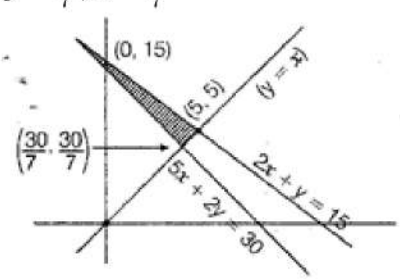
Section D

32. we have to minimise Z = 400x + 200y, subject to $5x + 2y \geqslant 30$

$2x + y \geqslant 15, x \leqslant y, x \geqslant 0, y \geqslant 0$.

On solving x - y = 0 and 5x + 2y = 30, we get

$y = \frac{30}{7}, x = \frac{30}{7}$



On solving x - y = 0 and 2x + y = 15, we get x = 5, y = 5

So, from the shaded feasible region it is clear that coordinates of corner points are (0, 15), (5, 5) and $\left(\frac{30}{7}, \frac{30}{7}\right)$.

Corner Points	**Corresponding Value of X = 400x + 200y**
(0, 15)	3000
(5, 5)	3000
$\left(\frac{30}{7}, \frac{30}{7}\right)$	$400 \times \frac{30}{7} + 200 \times \frac{30}{7} = \frac{18000}{7}$ = 2571.43 (minimum)

Hence, the minimum value is Rs 2571.43.

33. f is one-one: For any x, y $\in$ R - {+1}, we have $f(x) = f(y)$

$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$

$\Rightarrow$ xy + x = xy + y

$\Rightarrow$ x = y

Therefore, f is one-one function.

If f is one-one, let y = R - {1}, then f(x) = y

$\Rightarrow \frac{x}{x+1} = y$

$\Rightarrow x = \frac{y}{1-y}$

It is cleat that x $\in$ R for all y = R - {1}, also x = $\neq$ -1

Because x = -1

$\Rightarrow \frac{y}{1-y} = -1$

$\Rightarrow$ y = -1 + y

which is not possible.

Thus for each R - {1} there exists $x = \frac{y}{1-y} \in$ R - {1} such that

$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$

Therefore f is onto function.

OR

A = R – {3} and B = R – {1} and $f(x) = \frac{x-2}{x-3}$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1-2}{x_1-3}$ and $f(x_2) = \frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one function.

Now $y = \frac{x-2}{x-3}$

$\Rightarrow$ y(x - 3) = x - 2

$\Rightarrow$ xy - 3y = x - 2

$\Rightarrow$ x(y - 1) = 3y - 2

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{3y-2-3y+3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

34. Here,it is given equations of lines:

$L_1 : \frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$

$L_2 : \frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$

Direction ratios of L_1 and L_2 are (3, -1, 1) and (-3, 2, 4) respectively.

Suppose general point on line L_1 is P = (x_1, y_1, z_1)

$x_1 = 3s + 6$, $y_1 = -s + 7$, $z_1 = s + 4$

and suppose general point on line L_2 is Q = (x_2, y_2, z_2)

$x_2 = -3t$, $y_2 = 2t - 9$, $z_2 = 4t + 2$

$\therefore \overrightarrow{PQ} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$

$= (-3t - 3s - 6)i + (2t - 9 + s - 7)\hat{j} + (4t + 2 - s - 4)\hat{k}$

$\therefore \overrightarrow{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 16)\} + (4t - s - 2)\hat{k}$

Direction ratios of $\overrightarrow{PQ}$ are ((-3t - 3s - 6), (2t + s - 16), (4t - s - 2))

PQ will be the shortest distance if it is perpendicular to both the given lines

Thus, by the condition of perpendicularity,

$\Longrightarrow$ 3(-3t - 3s - 60) - 1(2t + s - 16) + 1(4t - s - 2) = 0 and

$\Longrightarrow$ -3(-3t - 3s - 6) + 2(2t + s - 16) + 4(4t - s - 2) = 0

$\Rightarrow$ -9t - 9s - 18 - 2t - s + 16 + 4t - s - 2 = 0 and

9t + 9s + 18 + 4t + 2s + - 32 + 16t - 4s - 8 = 0

$\Rightarrow$ -7t - 11s = 4 and

29t + 7s = -22

Solving above two equations,the we obtain

t = 1 and s = -1

therefore

P = (3, 8, 3) and Q = (-3, -7, 6)

Now, distance between points P and Q is

$d=\sqrt{(3+3)^2+(8+7)^2+(3-6)^2}$

$=\sqrt{(6)^2+(15)^2+(-3)^2}$

$=\sqrt{36+225+9}$

$=\sqrt{270}$

$=3\sqrt{30}$

Thus, the shortest distance between two given lines is

$d=3\sqrt{30}$ units

Now, the equation of the line passing through points P and Q is

$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$

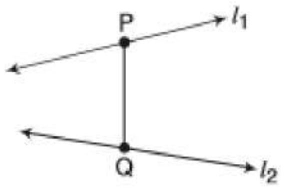
$\therefore \frac{x-3}{3+3}=\frac{y-8}{8+7}=\frac{z-3}{3-6}$

$\therefore \frac{x-3}{6}=\frac{y-8}{15}=\frac{z-3}{-3}$

$\therefore \frac{x-3}{2}=\frac{y-8}{5}=\frac{z-3}{-1}$ thus, the equation of the line of the shortest distance between two given lines is

$\frac{x-3}{2}=\frac{y-8}{5}=\frac{z-3}{-1}$

OR



Let P(3λ + 7, 2λ + 5, λ + 3) and

Q(2μ + 1, 4μ - 1, 3μ - 1)

Now, d.r'.s. of PQ = 3λ - 2μ + 6, 2λ - 4μ + 6, λ - 3μ + 4

According to question,

$\frac{3\lambda-2\mu+6}{2}=\frac{2\lambda-4\mu+6}{2}=\frac{\lambda-3\mu+4}{1}$

$\Rightarrow \lambda + 2\mu = 0$ and $2\mu = 2 \Rightarrow \mu = 1$

$\Rightarrow \lambda = -2\mu$

$\therefore \mu = 1, \lambda = -2$

$\therefore$ P(1, 1, 1) and Q(3, 3, 2)

PQ = $\sqrt{(3-1)^2+(3-1)^2+(2-1)^2}$

= $\sqrt{4+4+1}$

= 3

therefore length is 3 unit

Equation of PQ is $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-1}{1}$

35. We have, $f(x)=\begin{cases} x^2+3x+p, \, if\, x\leqslant 1 \\ qx+2, \, if\, x>1 \end{cases}$ is differentiable at x=1.

$\therefore Lf'(1)=\lim\limits_{x\to 1^-}\frac{f(x)-f(1)}{x-1}$

$=\lim\limits_{x\to 1^-}\frac{(x^2+3x+p)-(1+3+p)}{x-1}$

$=\lim\limits_{h\to 0}\frac{\left[(1-h)^2+3(1-h)+p\right]-[1+3+p]}{(1-h)-1}$

$=\lim\limits_{h\to 0}\frac{\left[1+h^2-2h+3-3h+p\right]-[4+p]}{-h}$

$=\lim\limits_{h\to 0}\frac{\left[h^2-5h+p+4-4-p\right]}{-h}=\lim\limits_{h\to 0}\frac{h[h-5]}{-h}$

$=-\lim\limits_{h\to 0}[h-5]=5$

$Rf'(1)=\lim\limits_{x\to 1^+}\frac{f(x)-f(1)}{x-1}=\lim\limits_{x\to 1^+}\frac{(qx+2)-(1+3+p)}{x-1}$

$=\lim\limits_{h\to 0}\frac{[q(1+h)+2]-(4+p)}{1+h-1}$

$= \lim_{h \to 0} \frac{[q+qh+2-4-p]}{h} = \lim_{h \to 0} \frac{qh+(q-2-p)}{h}$

Since f(x) is differentiable at x=1,therefore Rf'(1) must exist.For Rf'(x) to exist,we must have,

$q - 2 - p = 0 \Rightarrow p - q = -2$ ...(i)

$\therefore Rf'(1) = \lim_{h \to 0} \frac{qh+0}{h} = q$

Since f(x) is differentiable at x=1,therefore $Lf'(1) = Rf'(1)$, then 5 = q

From (1) $p - 5 = -2 \Rightarrow p = 3$

$\therefore p = 3$ and $q = 5$

Section E

36. **Read the text carefully and answer the questions:**

Mrs. Maya is the owner of a high-rise residential society having 50 apartments. When he set rent at ₹10000/month, all apartments are rented. If he increases rent by ₹250/ month, one fewer apartment is rented. The maintenance cost for each occupied unit is ₹500/month.

(i) If P is the rent price per apartment and N is the number of rented apartments, the profit is given by NP - 500 N = N(P - 500)

[∵ ₹500/month is the maintenance charge for each occupied unit]

(ii) Let R be the rent price per apartment and N is the number of rented apartments.

Now, if x be the number of non-rented apartments, then N(x) = 50 - x and R(x) = 10000 + 250x

Thus, profit = P(x) = NR = (50 - x) (10000 + 250 x - 500)

= (50 - x) (9500 + 250 x) = 250(50 - x) (38 + x)

(iii)We have, P(x) = 250(50 - x) (38 + x)

Now, P'(x) = 250[50 - x - (38 + x)] = 250[12 - 2x]

For maxima/minima, put P'(x) = 0

⇒ 12 - 2x = 0 ⇒ x = 6

Number of apartments are 6.

OR

P'(x) = 250(12 - 2x)

P''(x) = - 500 < 0

⇒ P(x) is maximum at x = 6

37. **Read the text carefully and answer the questions:**

In a city, there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys, the number of units of types I, II, and III respectively are 80, 70, and 65 in factory A and 85, 65, and 72 are in factory B. For girls the number of units of types I, II, and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

(i) In factory A, number of units of types I, II and III for boys are 80, 70, 65 respectively and for girls number of units of types I, II and III are 80, 75,90 respectively.

$$\therefore P = \begin{array}{c} \\ I \\ II \\ III \end{array} \begin{array}{c} \textit{Boys} \quad \textit{Girls} \\ \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix} \end{array}$$

In factory B, number of units of types I, II and III for boys are 85, 65, 72 respectively and for girls number of units of types I, II and III are 50, 55, 80 respectively.

$$Q = \begin{array}{c} \\ I \\ II \\ III \end{array} \begin{array}{c} \textit{Boys} \quad \textit{Girls} \\ \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} \end{array}$$

(ii) Let X be the matrix that represent the number of units of each type produced by factory A for boys and Y be the matrix that represents the number of units of each type produced by factory B for boys.

Then, $X = \overset{\quad I \quad\; II \quad III}{\begin{bmatrix} 80 & 70 & 65 \end{bmatrix}}$ and $Y = \overset{\quad I \quad\; II \quad III}{\begin{bmatrix} 85 & 65 & 72 \end{bmatrix}}$

Now, required matrix = X + Y = [80 70 65] + [80 65 72]

= [165 135 137]

(iii) Let X be the matrix that represent the number of units of each type produced by factory A for girls and Y be the matrix that represents the number of units of each type produced by factory B for girls.

$X = \overset{\quad I \quad\; II \quad III}{\begin{bmatrix} 80 & 75 & 90 \end{bmatrix}}$ and $Y = \overset{\quad I \quad\; II \quad III}{\begin{bmatrix} 50 & 55 & 80 \end{bmatrix}}$

Required matrix = [80 75 90] + [50 55 80]

= [130 130 137]

OR

Clearly, R = P + Q

$$= \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix} + \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} = \begin{bmatrix} 165 & 130 \\ 135 & 130 \\ 137 & 170 \end{bmatrix}$$

$$\therefore R' = \begin{bmatrix} 165 & 135 & 137 \\ 130 & 130 & 170 \end{bmatrix}$$

38. **Read the text carefully and answer the questions:**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(i) P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(Both are selected) = $P(A \cap B) = P(A) \cdot P(B)$ = $\frac{1}{3} . \frac{1}{2}$

P(Both are selected) = $\frac{1}{6}$

(ii) P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(none of them selected) = $P(A' \cap B') = P(A') \cdot P(B')$ = $\frac{2}{3} . \frac{1}{2}$

P(Both are selected) = $\frac{1}{3}$